Covariance Matrix Adaptation Evolution Strategy Using a Diversity-Guided Step-Size Tuning for Optimization in Electromagnetics

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Abstract — The covariance matrix adaptation evolution strategy (CMA-ES) is considered to be one of the most powerful and robust evolutionary algorithms for real-valued optimization with many successful applications to engineering problems. In this paper, the suitability of the classical CMA-ES and a novel CMA-ES using a diversity-guided step-size tuning (DCMA-ES) for electromagnetic design is tested on the TEAM workshop benchmark problem 22 and Loney's solenoid benchmark problem and results are compared with standard and advanced metaheuristics.

I. INTRODUCTION

The increasing need of highly efficient electromagnetic devices has inspired engineers to explore the performance and robustness of available optimization algorithms. Recent advances in bio-inspired metaheuristics, supported and encouraged by continually increasing power and speed of computers, make evolutionary algorithms, such as genetic algorithms, evolutionary programming, differential evolution, and evolution strategies and related techniques an attractive alternative for the optimization of electromagnetics devices.

In this context, some novel promising approaches related to Evolution Strategies (ES) have been recently proposed. While ES for real-valued optimization usually rely on Gaussian random variations. Appropriately adapting the covariance matrices of these mutations during optimization allows a form of learning and results in a variable metric for the search distribution. One of such techniques, the covariance matrix adaptation evolution strategy (CMA-ES) [1],[2] is considered state-of-the-art in ES.

In this paper, the performance of the classical CMA-ES and a novel CMA-ES using a diversity-guided step-size tuning (DCMA-ES) are tested on two well-known electromagnetic benchmark problems, namely the TEAM workshop benchmark problem 22 and Loney's solenoid benchmark problem. Furthermore, the performance of both methods is compared with that of other metaheuristics presented in the recent literature.

II. FUNDAMENTALS OF CMA-ES

The CMA-ES is an evolution strategy which adapts the full covariance matrix of a normal search (mutation) distribution. Compared to many other evolutionary algorithms, an important property of the CMA-ES is its invariance against linear transformations of the search space.

The $(\mu/\mu_w,\lambda)$-CMA-ES samples λ new candidate solutions and selects the μ best among them. These contribute in a weighted manner to the update of the distribution parameters. The algorithm is non-elitist by nature, but a practical implementation will preserve the best-ever evaluated solution. A detailed description of CMA-ES and the proposed DCMA-ES will be given in the extended version of the paper.

III. CASE STUDIES

A. Loney's solenoid design problem

Loney's solenoid design problem consists in determining the position and size of two correcting coils in order to generate a uniform magnetic flux density within a given interval on the axis of a main solenoid. The problem is described by two degrees of freedom (the separation s and the length l of the correcting coils) with box bounds (see Figure 1) [3].

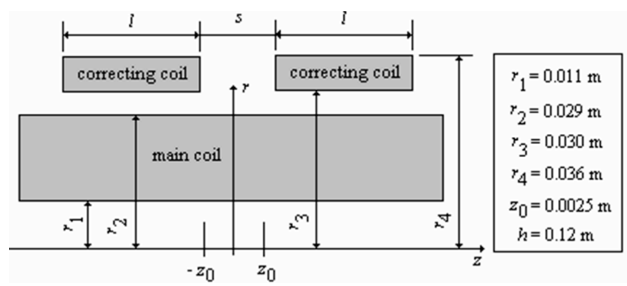

Fig. 1. Axial cross-section of Loney's solenoid (upper half-plane).

Three different basins of attraction of local minima can be recognized in the domain of F with values of $F > 4\cdot10^{-8}$ (high level region: HL), $3\cdot10^{-8} < F < 4\cdot10^{-8}$ (low level region: LL), and $F < 3\cdot10^{-8}$ (very low level region - global minimum region: VL). The very low level region is a small ellipsoidally shaped area within the thin low level valley.

B. TEAM workshop problem 22

The TEAM workshop problem 22 considers the optimal design of a superconducting magnetic energy storage (SMES) device in order to store a significant amount of energy in the magnetic field with a fairly simple and economical coil arrangement which can be rather easily scaled up in size. The benchmark consists in a continuous, constrained, eight-parameter problem, shown in Fig. 2, and further details can be found in [4].

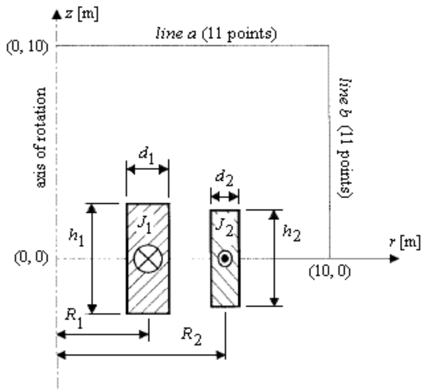

Fig. 2. Degrees of freedom of TEAM workshop problem 22.

It should be noted that here, as well as in [5], the objective function is defined as:

$$OF = \frac{B_{stray}^2}{B_{normal}^2} + w \cdot \frac{\left|Energy - E_{ref}\right|}{E_{ref}} \tag{1}$$

where the reference stored energy and stray field are E_{ref}= 180 MJ, B_{normal} = 200 μT, and w = 100 is a penalty factor which is used in order to make the stray field and energy terms error of roughly the same magnitude (this is a deviation from the original benchmark problem definition in which w=1.0). B_{stray}^2 in (3) is defined as:

$$B_{stray}^2 = \frac{\sum_{i=1}^{22}\left|B_{stray,i}\right|^2}{22} \tag{2}$$

where $B_{stray,i}$ is evaluated at 22 equidistant points along the lines *a* and *b* of Fig. 2.

IV. OPTIMIZATION RESULTS

The stopping criterion adopted was 1,500 and 4,000 objective function evaluations in each run for Loney's solenoid and the TEAM 22 benchmark, respectively. Furthermore, the λ candidate solutions in CMA-ES and DCMA-ES were set to 1, 10, 15, 20, 30, 40 and μ was kept to a constant value of 5 in the optimization results presented in Tables I to III. Table I and II also show results obtained with other algorithms as reported in [5] and [6], respectively.

It can be noted that while the best optimum of CMA-ES is in line with the best solutions found with other algorithms the mean and worst case as well as the standard deviation are not very satisfactory. On the other hand DCMA-ES can is competitive with or superior to other modern metaheuristics (PSO) especially for the higher-dimensional problem .

Furthermore, numerical results indicate that, for both benchmarks, an increase in λ does not correspond to better performance of the algorithm.

Since some of the results appear not to be up to the expectations further tuning of the CMA-ES and DCMA-ES algorithms are currently being performed and will be reported in the extended version of the paper.

TABLE I
SIMULATION RESULTS FOR THE LONEY'S SOLENOID OF F IN 30 RUNS

Optimization Method	$F(s, l)\cdot 10^{-8}$			
	Maximum (Worst)	Mean	Minimum (Best)	Standard Deviation
CMA-ES (5+10)	181.3400	22.0281	2.1995	38.5613
CMA-ES (5+15)	182.6919	24.5283	2.9372	45.7108
CMA-ES (5+20)	189.0848	24.4562	2.3063	45.3809
CMA-ES (5+30)	204.4167	25.4498	2.5812	49.9631
CMA-ES (5+40)	188.2613	23.5840	2.7557	44.7445
CMA-ES (1+15)	203.6377	24.0328	2.1074	49.5053
DCMA-ES (1+15)	27.2143	7.3751	**2.0571**	8.5328
Tribes (PSO) [5]	**3.9526**	**3.4870**	2.0574	**0.5079**

TABLE II
RESULTS (30 RUNS) FOR TEAM WORKSHOP PROBLEM 22

Optimization Method	Objective Function OF in 30 Runs			
	Maximum (Worst)	Mean	Minimum (Best)	Standard Deviation
CMA-ES (5+10)	103.4412	29.4580	0.1445	42.7100
CMA-ES (5+15)	103.4138	34.8530	0.3923	43.1912
CMA-ES (5+20)	103.4490	35.2021	0.2851	40.6165
CMA-ES (5+30)	104.7012	35.7487	0.7041	41.5678
CMA-ES (5+40)	103.5270	37.7371	3.8371	38.7113
CMA-ES (1+15)	103.5418	33.5382	0.5775	41.0632
DCMA-ES (1+15)	28.5601	**7.7832**	**0.2763**	6.4914
E-QPSO [6]	**26.1900**	7.9618	1.1730	**5.4340**

V. CONCLUSION

In this paper the performance of the standard CMA-ES and a novel DCMA-ES are tested on two well-known electromagnetic benchmark problems and compared with results obtained by other modern stochastic algorithms. The extended version will also include a thorough description of CMA-ES and DCMA-ES approaches and their implementation details.

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